Chapter

6 DIFFERENTIAL EQUATION

GROUP (A)-HOME WORK PROBLEMS

1) $\left(\frac{d^3y}{dx^3}\right)^{3/2} = \frac{xdy}{dx}$

Squaring

$\left(\frac{d^3y}{dx^3}\right)^3 = x^2\left(\frac{dy}{dx}\right)^2$

Order : 3
Degree : 3

2) $\sqrt{1 + \frac{1}{\left(dy/dx\right)^2}} = \left(\frac{d^2y}{dx^2}\right)^{3/2}$

Squaring

$1 + \frac{1}{\left(dy/dx\right)^2} = \left(\frac{d^2y}{dx^2}\right)^3$

Order : 2, Degree : 3.

3) $\left(y + \frac{dy}{dx}\right)^2 + x\frac{dy}{dx} = x^2$

$\therefore$ $y^2 + 2y\frac{dy}{dx} + \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2 = x^2$

Order : 1, Degree : 2.

4) $y\frac{dy}{dx} = \sqrt[3]{1 + \frac{d^2y}{dx^2}}$

Cubing

$y^3\left(\frac{dy}{dx}\right)^3 = 1 + \frac{d^2y}{dx^2}$

Order : 2, Degree : 1.

5) $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = K$

Order : 2, Degree : 1

6) $y = \frac{dy}{dx} + \left(1 + \frac{dy}{dx}\right)^{1/2}$

$\left(y - \frac{dy}{dx}\right)^2 = \left(1 + \frac{dy}{dx}\right)$

$y^2 - 2y\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{dy}{dx}$

Order : 1, Degree : 2

GROUP (B)-HOME WORK PROBLEMS

1) $y = c^2 + \frac{c}{x}$

Differentiating $y = c^2 + \frac{c}{x}$ ... (i)

w.r.t. x, we get, $\frac{dy}{dx} = -\frac{c}{x^2}$ $\therefore c = -x^2\frac{dy}{dx}$

Substituting this value of c in (i), we get,

$y = \left(-x^2\frac{dy}{dx}\right)^2 + \left(-\frac{x^2}{x}\cdot\frac{dy}{dx}\right)$

i.e., $y = x^4\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx}$.

This is the required D.E.

2) $y^2 - 2ay + x^2 = a^2$

Differentiating $y^2 - 2ay + x^2 = a^2$ ... (i)

w.r.t. x, we get, $2y\frac{dy}{dx} - 2a\frac{dy}{dx} + 2x = 0$

$\therefore$ $y\frac{dy}{dx} + x = a\frac{dy}{dx}$

Putting $\frac{dy}{dx} = y_1$, this becomes

$yy_1 + x = ay_1$ $\therefore a = \frac{yy_1 + x}{y_1}$

Substituting this value of a in (i), we get

$y^2 - 2y\left(\frac{yy_1 + x}{y_1}\right) + x^2 = \frac{(yy_1 + x)^2}{y_1^2}$

Multiplying throughout by y_1^2, this becomes

$$y^2y_1^2 - 2yy_1(yy_1 + x) + x^2y_1^2$$

$$= y^2y_1^2 + 2xyy_1 + x^2$$

$\therefore$ $(x^2 - 2y^2)y_1^2 - 4xyy_1 - x^2 = 0$

$\therefore$ $(x^2 - 2y^2)\left(\frac{dy}{dx}\right)^2 - 4xy\left(\frac{dy}{dx}\right) - x^2 = 0$

This is the required D.E.

3) $y = ae^x + be^{-x}$

$$\frac{dy}{dx} = ae^x - be^{-x}$$

$$\frac{d^2y}{dx^2} = ae^x + be^{-x}$$

$$\frac{d^2y}{dx^2} = y$$

$$\frac{d^2y}{dx^2} - y = 0$$

4) $y = ae^{-x} + b$

Differentiating $y = ae^{-x} + b$ ... (i)

twice w.r.t. x, we get,

$\frac{dy}{dx} = -ae^{-x}$ $\therefore \frac{d^2y}{dx^2} = ae^{-x} = -\frac{dy}{dx}$

$\therefore$ $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ is the required D.E.

Q-5) $y = Ae^{2x} + Be^{-2x}$

$$\frac{dy}{dx} = Ae^{2x}(2) + Be^{-2x}(-2)$$

$$\frac{dy}{dx} = 2Ae^{2x} - 2Be^{-2x}$$

$$\frac{d^2y}{dx^2} = 2Ae^{2x}(2) - 2Be^{-2x}(-2)$$

$$= 4Ae^{2x} + 4Be^{-2x}$$

$$\frac{d^2y}{dx^2} = 4y$$

$$\frac{d^2y}{dx^2} - 4y = 0$$

6) $xy = ae^x + be^{-x}$

Diff

$$x\frac{dy}{dx} + y = ae^x + be^{-x}(-1)$$

$$x\frac{dy}{dx} + y = ae^x - be^{-x}$$

Diff

$$\left[x\frac{d^2y}{dx^2} + \frac{dy}{dx}\right] + \frac{dy}{dx} = ae^x + be^{-x}$$

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = xy$$

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy = 0$$

7) $y = Ae^{5x} + Be^{-5x}$

$$\frac{dy}{dx} = 5Ae^{5x} - 5Be^{-5x}$$

$$\frac{d^2y}{dx^2} = 25Ae^{5x} + 25Be^{-5x}$$

$$= 25y$$

$$\frac{d^2y}{dx^2} - 25y = 0$$

8) $y = Ae^{5x+1} + Be^{-5x+1}$

$$\frac{dy}{dx} = Ae^{5x+1}(5) + Be^{-5x+1}(-5)$$

$$= 5Ae^{5x+1} - 5Be^{-5x+1}$$

$$\frac{d^2y}{dx^2} = 5Ae^{5x+1}(5) - 5Be^{-5x+1}(-5)$$

$$= 25Ae^{5x+1} + 25Be^{-5x+1}$$

$$= 25[Ae^{5x+1} + Be^{-5x+1}]$$

$$\frac{d^2y}{dx^2} = 25y$$

$$\frac{d^2y}{dx^2} - 25y = 0$$

9) $x = a\cos(\omega t + c)$

$$\frac{dx}{dt} = a[-\sin(\omega t + c)]\frac{d}{dt}(\omega t + c)$$

$$\frac{dx}{dt} = -a\sin(\omega t + c)[\omega(1) + 0]$$

$$= -a\omega\sin(\omega t + c)$$

$$\frac{d^2x}{dt^2} = -a\omega\cos(\omega t + c)\frac{d}{dt}[\omega t + 0]$$

$= -a\omega^2 \cos(\omega t + c)$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

10) $y = A \sin x + B \cos x$

$$\frac{dy}{dx} = A\cos x - B\sin x$$

$$\frac{d^2y}{dx^2} = -A\sin x - B\cos x$$

$$\frac{d^2y}{dx^2} = -(A\sin x + B\cos x)$$

$$\frac{d^2y}{dx^2} = -y$$

$$\frac{d^2y}{dx^2} + y = 0$$

11) $y = A \cos 3x + B \sin 3x$

$$\frac{dy}{dx} = -3A\sin 3x + 3B\cos 3x$$

$$\frac{d^2y}{dx^2} = -3A\cos 3x(3) + 3B(-\sin 3x)(3)$$

$= -9A\cos 3x - 9B \sin 3x$

$= -9[A\cos 3x + B\sin 3x]$

$= -9y$

12) $y = A\cos 4x + B\sin 4x$

Differentiating $y = A\cos 4x + B\sin 4x$ ... (i)

twice w.r.t. x, we get

$$\frac{dy}{dx} = -4A\sin 4x + 4B\cos 4x$$

$$\frac{d^2y}{dx^2} = -16A\cos 4x - 16B\sin 4x$$

$= -16(A\cos 4x + B\sin 4x) = -16y$ ... [By (i)]

$\therefore$ $\frac{d^2y}{dx^2} + 16y = 0$ is the required D.E.

13) $y = A \sin 2x + B \cos 2x$

$$\frac{dy}{dx} = 2A\cos 2x - 2B\sin 2x$$

$$\frac{d^2y}{dx^2} = -4A\sin 2x - 4B\cos 2x$$

$= -4(A\sin 2x + B\cos 2x)$

$= -4y$

14) $y = ae^{bx}$

$$\frac{dy}{dx} = ae^{bx}\frac{d}{dx}(bx)$$

$$\frac{dy}{dx} = ae^{bx}(b)(1)$$

$$\frac{dy}{dx} = abe^{bx}$$

$$\frac{dy}{dx} = b\left(ae^{bx}\right)$$

$$\frac{dy}{dx} = by$$

$$\frac{d^2y}{dx^2} = b\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} - b\frac{dy}{dx} = 0$$

15) $y = ae^{2x} + be^{-3x}$ ... (i)

$$\frac{dy}{dx} = 2ae^{2x} - 3be^{-3x} \quad \text{... (ii)}$$

$$\frac{d^2y}{dx^2} = 4ae^{2x} + 9be^{-3x} \quad \text{... (iii)}$$

Equation (i) (ii) (iii) are consistent

$$\begin{vmatrix} y & 1 & 1 \\ \frac{dy}{dx} & 2 & -3 \\ \frac{d^2y}{dx^2} & 4 & 9 \end{vmatrix} = 0$$

$y(15+12) - 1$

$$\left(9\frac{dy}{dx} + 3\frac{d^2y}{dx^2}\right) + 1\left(4\frac{dy}{dx} - 2\frac{d^2y}{dx^2}\right) = 0$$

$$30y - 5\frac{dy}{dx} - 5\frac{d^2y}{dx^2} = 0$$

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

16) $y = (c_1 + c_2x)e^x$

$y = (c_1 + c_2x)e^x$

$\therefore$ $e^{-x}y = c_1 + c_2x$

Differentiating w.r.t. x, we get,

$$e^{-x}.\frac{dy}{dx} - e^{-x}.y = c_2 \quad \therefore e^{-x}\left(\frac{dy}{dx} - y\right) = c_2$$

Differentiating again w.r.t. x, we get,

$$e^{-x}\left(\frac{d^2y}{dx^2}-\frac{dy}{dx}\right)-e^{-x}\left(\frac{dy}{dx}-y\right)=0$$

$$\therefore \quad e^{-x}\left(\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y\right)=0$$

$$\therefore \quad \frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$$

This is the required D.E.

17) $y = C_1 e^{2x} + C_2 e^{-x}$ ... (i)

$$\frac{dy}{dx}=C_1(2)e^{2x}-C_2e^{-x}$$

$$\frac{dy}{dx}=2C_1e^{2x}-C_2e^{-x} \quad \text{... (ii)}$$

$$\frac{d^2y}{dx^2}=4C_1e^{2x}+C_2e^{-x} \quad \text{... (iii)}$$

$$\begin{vmatrix} y & 1 & 1 \\ \frac{dy}{dx} & 2 & 1 \\ \frac{d^2y}{dx^2} & 4 & 1 \end{vmatrix}=0$$

$$y\,(2+4)-1\left(\frac{dy}{dx}+\frac{d^2y}{dx^2}\right)+\left(4\frac{dy}{dx}+-2\frac{d^2y}{dx^2}\right)=0$$

$$6y+3\frac{dy}{dx}-3\frac{d^2y}{dx^2}=0$$

$$\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=0$$

18) $y = Ae^{3x}+Be^{-2x}$

$$y = Ae^{3x}+Be^{-2x} \quad \text{... (i)}$$

Differentiating twice w.r.t. x, we get,

$$\frac{dy}{dx}=3Ae^{3x}-2Be^{-2x} \quad \text{... (ii)}$$

$$\frac{d^2y}{dx^2}=9Ae^{3x}+4Be^{-2x} \quad \text{... (iii)}$$

These three equations in Ae^{3x} and Be^{-2x} are consistent.

∴ determinant of their consistency condition is zero.

$$\therefore \quad \begin{vmatrix} y & 1 & 1 \\ dy/dx & 3 & -2 \\ d^2y/dx^2 & 9 & 4 \end{vmatrix}=0$$

$$\therefore \quad y(12+18)-\frac{dy}{dx}(4-9)+\frac{d^2y}{dx^2}(-2-3)=0$$

$$\therefore \quad 30y+5\frac{dy}{dx}-5\frac{d^2y}{dx^2}=0$$

$$\therefore \quad \frac{d^2y}{dx^2}-\frac{dy}{dx}-6y=0$$

This is the required D.E.

Alternative Method :

$$y = Ae^{3x}+Be^{-2x}$$

Dividing both sides by e^{-2x}, we get,

$$e^{2x}y = Ae^{5x}+B$$

Differentiating w.r.t. x, we get,

$$e^{2x}\frac{dy}{dx}+y\,.\,e^{2x}\,.\,2=Ae^{5x}\,.\,5+0$$

$$\therefore \quad e^{2x}\left(\frac{dy}{dx}+2y\right)=5Ae^{5x}$$

Dividing both sides by e^{5x}, we get,

$$e^{-3x}\left(\frac{dy}{dx}+2y\right)=5A$$

Differentiating again w.r.t. x, we get,

$$e^{-3x}\left(\frac{d^2y}{dx^2}+2\frac{dy}{dx}\right)+\left(\frac{dy}{dx}+2y\right).\,e^{-3x}(-3)=0$$

$$\therefore \quad e^{-3x}\left(\frac{d^2y}{dx^2}+2\frac{dy}{dx}-3\frac{dy}{dx}-6y\right)=0$$

$$\therefore \quad \frac{d^2y}{dx^2}-\frac{dy}{dx}-6y=0$$

This is the required D.E.

19) y = ex (a cos x + b sin x)

ye^{-x} = a cos x + b sin x

$$y\frac{d}{dx}\left(e^{-x}\right)+e^{-x}\frac{dy}{dx} = a(-\sin x)+b\cos x$$

$$ye^{-x}(-1)+e^{-x}\frac{dy}{dx}=-a\sin x+b\cos x$$

$$e^{-x}\left(\frac{dy}{dx}-y\right)=-a\sin x+b\cos x$$

$$e^{-x}\left[\frac{d^2y}{dx^2}-\frac{dy}{dx}\right]+\left(\frac{dy}{dx}-y\right)(e^{-x})(-1)$$

$= -a\cos x + b(-\sin x)$

$$\left[\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y\right](ye^{-x})$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

20) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Differentiating $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ... (i)

w.r.t. x, we get,

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0 \quad \therefore y\frac{dy}{dx} = -\frac{b^2}{a^2}x$$ ... (ii)

Differentiating again w.r.t. x, we get,

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -\frac{b^2}{a^2}$$

Substituting the value of $-(b^2/a^2)$ in (ii) we get,

$$y\frac{dy}{dx} = x\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right]$$

$$\therefore \quad xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

This is the required D.E.

21) $(y-a)^2 = 4(x-b)$

The given equation is $(y-a)^2 = 4(x-b)$ ... (i)

Differentiating twice w.r.t. x, we get,

$$2(y-a)\frac{dy}{dx} = 4, \text{ i.e., } (y-a)\frac{dy}{dx} = 2 \quad \text{... (ii)}$$

and $(y-a)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ ... (iii)

From (ii), $y - a = \frac{2}{dy/dx}$

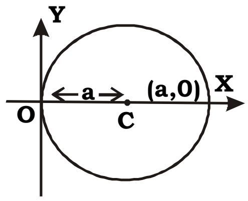
Substituting this value in (iii), we get,

$$\frac{2}{dy/dx}\cdot\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

$$\therefore \quad 2\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$$

This is the required D.E.

22) Form the differential equation of the family of circles which pass through the origin and whose centres lie on the X-axis. The equation of a circle passing through the origin and having centre on the X-axis is



$$(x-a)^2 + y^2 = a^2$$

i.e., $x^2 + y^2 = 2ax$ ... (i)

where a is an arbitary constant

Differentiating w.r.t. x, we get,

$$2x + 2y\frac{dy}{dx} = 2a$$

Substituting this value of 2a in (i), we get,

$$x^2 + y^2 = x\left(2x + 2y\frac{dy}{dx}\right),$$

i.e., $y^2 - x^2 = 2xy\frac{dy}{dx}$

This is the D.E. of the family of circles

GROUP (C)-HOME WORK PROBLEMS

1) $y = \frac{x}{(x+1)}$

Diff w.r.t x

$$\therefore \quad (x+1)\frac{dy}{dx} + y = 1$$

Hence $y = \frac{x}{x+1}$ is the solution of

$$(x+1)\frac{dy}{dx} + y = 1.$$

2) $y = ae^{2x}$

$$\therefore \quad \frac{dy}{dx} = 2ae^{2x}$$

$$\therefore \quad \frac{d^2y}{dx^2} = 4ae^{2x}$$

$$\therefore \quad \frac{d^2y}{dx^2} = 4y$$

3) $y = 2\sin 3x$

$$\frac{dy}{dx} = 2(3)\cos 3x$$

$\therefore \quad \frac{d^2y}{dx^2} = -9(2 \sin 3x)$

$\therefore \quad \frac{d^2y}{dx^2} + 9y = 0$

4) $y = a + \frac{b}{x}$

$\therefore \quad \frac{dy}{dx} = 0 - \frac{b}{x^2}$

$\therefore \quad x^2 \frac{dy}{dx} = -b$

$\therefore \quad x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} = 0$

5) $y = ae^{2x}$

$\frac{dy}{dx} = 2ae^{2x}$

$\therefore \quad \frac{dy}{dx} = 2y$

$y = ae^{2x}$

$\frac{y}{a} = e^{2x}$

$\log(y \backslash a) = 2x$

$\frac{(\log y - \log a)}{x} = 2$

$\frac{dy}{dx} = \left(\frac{\log y - \log a}{x}\right).y$

$x \frac{dy}{dx} - \log y \, . \, y = 0$

When $x = 1, y = e$

$e = ae^2$

$a = \frac{e}{e^2} = e^{-1}$

$a = \frac{1}{e}$

6) $y = (\sin^{-1}x)^2 + c$

$\frac{dy}{dx} = \frac{2 \sin^{-1} x}{\sqrt{1 - x^2}}$

$\left(\sqrt{1 - x^2}\right) \frac{dy}{dx} = 2\sin^{-1}x$

$\left(\sqrt{1 - x^2}\right) \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{(-2x)}{2\sqrt{1 - x^2}} = \frac{2}{\sqrt{1 - x^2}}$

$\therefore \quad (1 - x^2) \frac{d^2y}{dx^2} - \frac{xdy}{dx} = 2$

7) $y = x^2e^x$

$\frac{dy}{dx} = x^2e^x + 2xe^x$

$\frac{d^2y}{dx^2} = x^2e^x + e^x(2x) + 2e^x + 2xe^x$

R.H.S. $= \frac{1}{y} \left(\frac{dy}{dx}\right)^2$

$= \frac{1}{x^2e^x} (x^2e^x + 2xe^x)^2$

$= \frac{(x^2e^x)^2 + (4x^3e^{2x}) + 4x^2e^{2x}}{x^2e^x}$

$= x^2e^x + 4xe^x + 2xe^x$

$= \frac{d^2y}{dx^2}$ = LHS.

Hence verified that $y = x^2e^x$ is the solution of $\frac{d^2y}{dx^2} = \frac{1}{y} \left(\frac{dy}{dx}\right)^2$

8) $y = \sin x$

$\frac{dy}{dx} = \cos x, \quad \frac{d^2y}{dx^2} = -\sin x$

L.H.S. $= \left(\frac{d^2y}{dx^2} - y\right)^2$

$= (-\sin x - \sin x)^2 = 4 \sin^2 x$

R.H.S. $= 4\left(1 - \left(\frac{dy}{dx}\right)^2\right)$

$= 4 (1 - \cos^2 x)$

$= 4\sin^2 x$

Hence verified that $y = \sin x$ is a solution of

$\left(\frac{d^2y}{dx^2} - y\right)^2 = 4 \left[1 - \left(\frac{dy}{dx}\right)^2\right]$

GROUP (D)-HOME WORK PROBLEMS

1) $y(1 + \log x)dx - x\log x\, dy = 0.$

$\therefore \quad \dfrac{(1 + \log x)dx}{x \log x} = \dfrac{dy}{y}$

Intergrating both sides

$\log|x \log x| = \log|y| + \log|c|$

$\therefore \quad \log|x \log x| = \log|cy|$

$\therefore \quad x \log x = cy$

2) $e^y \cos x\, dy + (e^y + 1) \sin x\, dx = 0.$

$\therefore \quad \int \dfrac{e^y}{e^y + 1} dy = \int \dfrac{-\sin x\, dx}{\cos x}$

$\therefore \quad \log|e^y + 1| = \log|\cos x| + \log c$

$\therefore \quad e^y + 1 = c(\cos x)$

3) $e^x(1 + y^2)dx + (e^x + 1)dy = 0$

$\therefore \quad \int \dfrac{e^x dx}{e^x + 1} + \int \dfrac{dy}{1 + y^2} = 0$

$\therefore \quad \log|e^x + 1| + \tan^{-1} y = c$

4) $e^x(y + 1)\, dx - (e^x + 1)\, dy = 0$

$\therefore \quad \int \dfrac{e^x}{e^x + 1} dx = \int \dfrac{dy}{(y + 1)}$

$\therefore \quad \log|e^x + 1| = \log|y + 1| + \log c.$

$\therefore \quad e^x + 1 = c(y + 1)$

5) $\int (e^x + x^2)\, dx + \int e^{-y}\, dy = 0$

$\therefore \quad e^x + \dfrac{x^3}{3} - e^{-y} = c$

$\therefore \quad (e^x - e^{-y}) + \dfrac{x^3}{3} = c.$

6) $dx + x\, dy = 0$

$\therefore \quad \int \dfrac{dx}{x} + \int dy = 0$

$\therefore \quad \log|x| + y = c$

7) $\int e^x (\sin x + \cos x)\, dx = \int -e^y (\cos y - \sin y)\, dy$

$\therefore \quad e^x \sin x = -e^y \cos y + c$

$\therefore \quad e^x \sin x + e^y \cos y = c$

8) $e^x dx + (e^x - 1) \sin^2 y\, dy = 0$

$\int \dfrac{e^x}{e^x - 1} dx + \int \dfrac{(1 - \cos 2y)}{2} dy = 0$

$\log|e^x - 1| + \dfrac{y}{2} - \dfrac{\sin 2y}{4} = c$

9) $\dfrac{dy}{dx} + \sqrt{\dfrac{1 - y^2}{1 - x^2}} = 0$

$\therefore \quad \int \dfrac{dx}{\sqrt{1 - x^2}} + \int \dfrac{dy}{\sqrt{1 - y^2}} = 0$

$\therefore \quad \sin^{-1} x + \sin^{-1} y = c.$

10) $\int (y \cos y + \sin y)\, dy = \int (x + 2x \log x)\, dx$

$\therefore \quad y \sin y - \int \sin y\, (1)\, dy + \int \sin y\, dy$

$= \dfrac{x^2}{2} + 2 \log x \dfrac{x^2}{2} - 2\int \dfrac{x^2}{2x} dx$

$\therefore \quad y \sin y = \dfrac{x^2}{2} + x^2 \log x - \dfrac{x^2}{2} + c$

$\therefore \quad y \sin y = x^2 \log x + c.$

11) $(1 + x^2)\, y dy + (1 + y^2)\, x dx = 0$

$\therefore \quad \int \dfrac{2y\, dy}{(1 + y^2)} = -\int \dfrac{2x\, dx}{(1 + x^2)}$

(Multiplying both sides by 2)

$\therefore \quad \log|(1 + y^2)| = -\log|(1 + x^2)| + \log|c|$

$\therefore \quad (1 + y^2)(1 + x^2) = c$

12) $x dx + y\, dy = 0$

$\therefore \quad \int x dx + \int y dy = 0$

$\therefore \quad \dfrac{x^2}{2} + \dfrac{y^2}{2} = c$

$\therefore \quad x^2 + y^2 = 2c$

$\therefore \quad x^2 + y^2 = c$

13) $y^2 dx + x^2 dy = 0$

$\therefore \quad \dfrac{dx}{x^2} = \dfrac{-dy}{y^2}$

$\therefore \quad \dfrac{-1}{x} = \dfrac{1}{y} + c$

$\therefore \quad \dfrac{1}{x} + \dfrac{1}{y} = c$

$c = \left(\dfrac{1}{x} + \dfrac{1}{y}\right)$

14) $(1 - x)dy - (1 + y)\, dx = 0.$

$\therefore \quad \int \dfrac{dy}{(1 + y)} = \int \dfrac{dx}{(1 - x)}$

$\therefore \quad \log|1 + y| = -\log|1 - x| + \log c$

$\therefore \quad (1 + y)\, (1 - x) = c$

15) $x(1+y)\,dy - y(x+1)dx = 0$

$\therefore \int \frac{(1+y)}{y}dy = \int \frac{(x+1)}{x}dx$

$\therefore \log|y| + y = x + \log|x| + \log|c|$

$\therefore y = x + \log\left|\frac{xc}{y}\right|$

16) $x(1+y^2)dx + y(1+x^2)dy = 0$

$\therefore \int \frac{2x\,dx}{(1+x^2)} = \int \frac{-2y\,dy}{(1+y^2)}$

$\therefore \log|1+x^2| = -\log|1+y^2| + \log|c|$

$\therefore (1+x^2)(1+y^2) = c$

GROUP (F)-HOME WORK PROBLEMS

1) $\frac{dy}{dx} = (2x - 3y + 1)^2$

$\therefore 2x - 3y + 1 = u$

$\therefore 2 - 3\frac{dy}{dx} = \frac{du}{dx}$

$\therefore \frac{dy}{dx} = \frac{-1}{3}\left(\frac{du}{dx} - 2\right)$

$\therefore \frac{-1}{3}\left(\frac{du}{dx} - 2\right) = u^2$

$\therefore \frac{du}{dx} - 2 = -3u^2$

$\therefore \int \frac{du}{-3u^2 + 2} = \int -dx$

$\frac{1}{2(\sqrt{2})}\log\left|\frac{\sqrt{2}+\sqrt{3}u}{\sqrt{2}-\sqrt{3}u}\right| = -x + c$

$\therefore \frac{1}{2(\sqrt{2})}\log\left|\frac{\sqrt{2}+\sqrt{3}(2x-3y+1)}{\sqrt{2}-\sqrt{3}(2x-3y+1)}\right| = -x + c$

2) $y = ux$

$\frac{dy}{dx} = u + x\frac{du}{dx}$

$\therefore u + x\frac{du}{dx} = \frac{ux + \sqrt{x^2 + u^2x^2}}{x}$

$\therefore u + x\frac{du}{dx} = u + \sqrt{1+u^2}$

$\therefore x\frac{du}{dx} = \sqrt{1+u^2}$

$\therefore \int \frac{du}{\sqrt{1+u^2}} = \int \frac{dx}{x}$

Put $u = \tan\theta$ $\quad du = \sec^2\theta d\theta$

$\therefore \int \frac{\sec^2\theta d\theta}{\sec\theta} = \int \frac{dx}{x}$

$\therefore \log|\sec\theta + \tan\theta| = \log|x| + \log c$

$\therefore \log\left|u + \sqrt{1+u^2}\right| = \log|x| + c.$

3) $\left(x\frac{dy}{dx} - y\right)\sin\left(\frac{y}{x}\right) = x^2e^x.$

$y = ux$

$\frac{dy}{dx} = u + x\frac{du}{dx}$

$\left(x\left(u + x\frac{du}{dx}\right) - ux\right)\sin u = x^2e^x$

$\therefore x^2\frac{du}{dx}\sin u = x^2e^x$

$\therefore \int \sin u\,du = \int e^x$

$\therefore -\cos u = e^x + c$

$\therefore -\cos\frac{y}{x} = e^x + c$

4) $1 + \frac{dy}{dx} = \text{cosec}\,(x+y)$

$x + y = u$

$1 + \frac{dy}{dx} = \frac{du}{dx}$

$\therefore \frac{du}{dx} = \text{cosec}\,u$

$\therefore \int \sin u\,du = \int dx$

$\therefore \cos u = x + c$

$\therefore \cos(x+y) = x + c.$

5) $(2x - 3y)\left(2 - 3\frac{dy}{dx}\right) = e^{-x}$

$2x - 3y = u$

$\therefore 2 - 3\frac{dy}{dx} = \frac{du}{dx}$

$\therefore u\frac{du}{dx} = e^{-x}$

$\therefore \int u\,du = \int e^{-x}dx$

$\therefore \frac{u^2}{2} = -e^{-x} + c.$

$\frac{(2x-3y)^2}{2} = -e^x + c$

GROUP (G)-HOME WORK PROBLEMS

1) $x^2 \frac{dy}{dx} = xy + y^2$

This is a homogeneous Differential equatiion

Put y = vx

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore \quad x^2\left(v + x\frac{dv}{dx}\right) = x(vx) + v^2x^2$

$\therefore \quad x^2\left(v + x\frac{dv}{dx}\right) = x^2(v^2 + v)$

$\therefore \quad x\frac{dv}{dx} = v^2$

$\therefore \quad \int\frac{dv}{v^2} = \int\frac{dx}{x}$

$\therefore \quad \frac{-1}{v} = \log|x| + c$

$\therefore \quad \frac{-x}{y} = \log|x| + c$

2) $\left(x\frac{dy}{dx} - y\right)e^{\frac{y}{x}} = x$

y = vx

$\left(\frac{dy}{dx} = v + x\frac{dv}{dx}\right)$

$\left[x\left(v + x\frac{dv}{dx}\right) - (vx)\right]e^v = x$

$\therefore \quad \left[xv + x^2\frac{dv}{dx} - vx\right]e^v = x$

$\therefore \quad x^2\frac{dv}{dx}e^v = x$

$\therefore \quad \int e^v dv = \int\frac{dx}{x}$

$\therefore \quad e^v = \log|x| + c$

$\therefore \quad e^{y/x} = \log|x| + c.$

3) $(x^2 + 3xy + y^2)\,dx - x^2dy = 0$

$\therefore \quad \frac{dy}{dx} = \left(\frac{x^2 + 3xy + y^2}{x^2}\right)$

y = vx

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

This is a homogeneous Differential equatiion

Put y = vx

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$x\frac{dv}{dx} + v = \frac{x^2 + 3xy + v^2x^2}{x^2}$

$\therefore \quad x\frac{dv}{dx} + v = 1 + 3v + v^2$

$\therefore \quad x\frac{dv}{dx} = 1 + 2v + v^2$

$\therefore \quad \int\frac{dv}{(1+v)^2} = \int\frac{dx}{x}$

$\therefore \quad \frac{-1}{1+v} = \log x + c$

$\therefore \quad \frac{-1}{1+\frac{y}{x}} = \log x + c$

$\therefore \quad \frac{-x}{x+y} = \log x + c$

4) $x\sin\frac{y}{x}\,dy + x - y\sin\frac{y}{x}\,dx = 0$

$\therefore \quad \frac{dy}{dx} = \frac{-x + y\sin\left(\frac{y}{x}\right)}{x\sin\left(\frac{y}{x}\right)}$

y = vx

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

This is a homogeneous Differential equatiion

Put y = v x

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore \quad x\frac{dv}{dx} + v = \frac{-x + vx\sin v}{x\sin v}$

$\therefore \quad x\frac{dv}{dx} + v = \frac{-1 + v\sin v}{\sin v}$

$\therefore \quad x\frac{dv}{dx} + v = \frac{-1}{\sin v} + v$

$\therefore \quad \int\sin v\,dv = \int\frac{dx}{x}$

$\therefore \quad \cos v = \log|x| + c$

$\therefore \quad \cos\frac{y}{x} = \log|x| + c$

5) $x\frac{dy}{dx} = y(\log y - \log x)$

This is a homogeneous Differential equatiion

Put y = vx

$\frac{dy}{dx} = v + x\ \frac{dv}{dx}$

$x\left(x\frac{dv}{dx} + v\right) = vx\left(\log\ \frac{vx}{x}\right)$

$\therefore\quad x^2\frac{dv}{dx} + xv = xv\ (\log v)$

$\therefore\quad x^2\frac{dv}{dx} + v = v\log v$

$\therefore\quad x\ \frac{dv}{dx} = v\ (\log v - 1)$

$\therefore\quad \int\frac{dv}{v(\log v - 1)} = \int\frac{dx}{x}$

$\therefore\quad \log|\log v - 1| = \log|x| + \log c$

$\therefore\quad \frac{\log v - 1}{x} = C$

$\therefore\quad \log\frac{y}{x} = cx + 1$

6) $(x + y)\ \frac{dy}{dx} + y = 0$

This is a homogeneous Differential equatiion

Put y = vx

$\frac{dy}{dx} = v + x\ \frac{dv}{dx}$

$\therefore\quad (x + vx)\left(v + \frac{xdv}{dx}\right) + vx = 0$

$\therefore\quad x(1 + v)\left(v + \frac{xdv}{dx}\right) + vx = 0$

$\therefore\quad x\left[(1+v)\left(v + \frac{xdv}{dx}\right) + v\right] = 0$

$x\left[v + v\frac{xdv}{dx} + \frac{xdv}{dx} + v^2 + v\right] = 0$

$\therefore\quad x\left[2v + v^2 + x(1+v)\frac{dv}{dx}\right] = 0$

$\therefore\quad 2v + v^2 = -x(1+v)\frac{dv}{dx}$

$\therefore\quad \frac{2v(1+v)}{(1+v)} = -x.\frac{dv}{dx}$

$\therefore\quad 2\times\frac{1}{x}dx = \frac{-1}{v}dv$

$2\log|x| = \log|v| + \log c$

$2\log|x| = -\log|v| + \log c$

$2\log|x| + \log|v| = \log c$

$x^2 . v = c$

$2x^2.\frac{y}{x} = c$

$c = 2xy$

7) $\frac{dy}{dx} = e^{y/x} + \frac{y}{x}$

$\therefore\quad v + x\frac{dv}{dx} = e^v + v$

$\therefore\quad x\frac{dv}{dx} = e^v$

$\therefore\quad \int e^{-v}dv = \int\frac{dx}{x}$

$\therefore\quad -e^{-y} = \log|x| + c$

$\therefore\quad e^{-y/x} + \log|x| = c$

8) $\frac{dy}{dx} = \frac{x + y}{x - y}$

$\therefore\quad v + x\frac{dv}{dx} = \frac{x + vx}{x - vx}$

$\therefore\quad v + x\frac{dv}{dx} = \frac{x(1 + v)}{x(1 - v)}$

$\therefore\quad x\frac{dv}{dx} = \frac{1 + v}{1 - v} - v$

$\therefore\quad x\frac{dv}{dx} = \frac{1 + v - v + v^2}{1 - v}$

$\therefore\quad x\frac{dv}{dx} = \frac{1 + v^2}{1 - v}$

$\therefore\quad \int\frac{1 - v}{1 + v^2}dv = \int\frac{dx}{x}$

$\therefore\quad \tan^{-1}(v) - \frac{1}{2}\log|1 + v^2| = \log|x| + c$

$\therefore\quad 2\tan^{-1}\left(\frac{y}{x}\right) = \log\left|\left(1 + \frac{y^2}{x^2}\right)x^2\right| + c$

$\therefore\quad 2\tan^{-1}\left(\frac{y}{x}\right) = \log|x^2 + y^2| + c$

9) $x\frac{dy}{dx} = x + 2y$

$\therefore \quad x\left(v + x\frac{dv}{dx}\right) = x + 2xv$

$\therefore \quad xv + x^2\frac{dv}{dx} = x + 2xv$

$\therefore \quad x^2\frac{dv}{dx} = x + xv \quad \Rightarrow \quad x\frac{dv}{dx} = 1 + v$

$\therefore \quad \int\frac{dv}{1 + v} = \int\frac{dx}{x}$

$\therefore \quad \log(1 + v) = \log|x| + \log|c|$

$\therefore \quad \log\left|\frac{1 + \frac{y}{x}}{x}\right| = \log|c|$

$\therefore \quad x + y = cx^2$